D.S.(II) Maths
paper-III, Group-B.
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Intinkoon's Method of finding the auxiliary series

We have seen in the comparison test that if $\lim_{n\to\infty} \frac{u_n}{v_n} = l$ where l is non-zero finite greater then the two $\sum u_n$ and $\sum v_n$ either converge or diverges. Hence we infer that the convergency or divergency of $\sum u_n$ depends upon the convergency or divergency of $\sum v_n$, i.e. if $\sum v_n$ is convergent then $\sum u_n$ will also be convergent and if $\sum v_n$ is divergent then the series $\sum u_n$ will also be divergent.

The series $\sum v_n$ is called an Auxiliary series. To find the convergency or divergency of a series whose nth term is u_n, we must know the nature of the auxiliary series $\sum v_n$. A method of finding v_n is given below.

Let us take u_n, the nth term of the given series and retain only the highest powers of n in both numerator and denominator. Let us denote this result by v_n and

$$u_n = \text{a constant} + \text{some powers of } n.$$

Ex 1 consider $u_n = \frac{n^2+1}{n^3+2}$

Here, the term containing the highest power of n in the numerator $= n^2$ and in the denominator $= n^3$

$\therefore$ we take $v_n = \frac{n^2}{n^3} = \frac{1}{n}$

[illegible]

[illegible]

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[illegible]

[illegible]

③ Let us consider an auxiliary series $\sum v_n$ whose nth term v_n is $\frac{1}{n}$

$$\therefore \frac{u_n}{v_n} = \frac{2-1/n}{2-1/n+1/n^2}$$

Now taking limit when $n \to \infty$, we have

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \frac{2-1/n}{2-1/n+1/n^2} = \frac{2}{2} = 1 \text{ finite and non-zero}$$

By comparison test, both the series $\sum u_n$ and $\sum v_n$ will be of the same nature. Here $\sum v_n$ is divergent, so $\sum u_n$ is also divergent.

Prove that the series

$1+\frac{4}{6}+\frac{6}{10}+\frac{8}{17}+\cdots+\frac{2n}{n^2+1}+\cdots$ is divergent

Sol. Here $u_n=\frac{2n}{n^2+1}=\frac{2n}{n^2(1+1/n^2)}=\frac{2}{n(1+1/n^2)}$

Let us take an auxiliary series whose nth term v_n is $1/n$

$\therefore \frac{u_n}{v_n}=\frac{2}{n(1+1/n^2)}\times\frac{n}{1}=\frac{2}{1+1/n^2}$

$\therefore \lim_{n\to\infty}\frac{u_n}{v_n}=\lim_{n\to\infty}\frac{2}{1+1/n^2}=2=$ a finite quantity

By comparison test both the series $\sum u_n$ and $\sum v_n$ will be of the same nature

But the series $\sum v_n$ whose nth term is $v_n=1/n$ is divergent, so the given series $\sum u_n$ is also divergent

(3) Examine the series

$1+3/4+\frac{5}{13}+\frac{7}{25}+\cdots+\frac{2n-1}{2n^2-2n+1}+\cdots$

Sol. Here, we have

$u_n=\frac{2n-1}{2n^2-2n+1}=\frac{1}{n}\frac{(2-1/n)}{(2-\frac{2}{n}+1/n^2)}$